

# Finding the Probability Distribution for Failures

*This article shows that the feasibility of scheduled discard as a failure management strategy mainly depends on the failure's probability distribution. Although the term scheduled discard is used throughout this paper, please read it as scheduled discard and/or scheduled restoration.*

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The article discusses on the importance of technical history data, considers the differences between age-related failures and random failures and it demonstrates the role of statistics using the normal probability distribution, the exponential probability distribution and the Weibull probability distribution as examples. The article also illustrates that, from a philosophical point of view, we should formulate our theories as falsifiable hypotheses. In this article the author defends his view that the hypothesis that the failure conforms to the exponential probability distribution, should be selected as the null hypothesis. Finally it shows how that null hypothesis can be tested.

## The Importance of Technical History Data

We are intrigued by data about failures that occurred in the past. We are eager to learn from these data. We want to distil information in order to adapt our strategy. It would for instance be helpful to know which failures have occurred in the past, how often they occurred, what happened when they occurred, how they were handled etc.

This paper focuses on the following question. Can we tell how long it will take before a specific failure will occur again, using technical history data about that failure? That information would enable us to anticipate on the occurrence of that failure. As an example, imagine that a specific failure comes back every so and so many years, months, weeks, running hours, kilometres etc. In that case we would seriously consider discarding the associated component just before that failure would occur. A colleague of the author puts it very explicit: "You want

to do it to the machine before the machine does it to you".

According to SAE Standard JA1011 "Evaluation Criteria for Reliability-Centered Maintenance (RCM) Processes", scheduled discard is a feasible failure management strategy only if there is a clearly defined age at which there is an increase in the conditional\* probability of the failure mode under consideration. Such failures are known as age-related failures. This means that SAE Standard JA1011 also implies that scheduled discard is not a feasible failure manage-

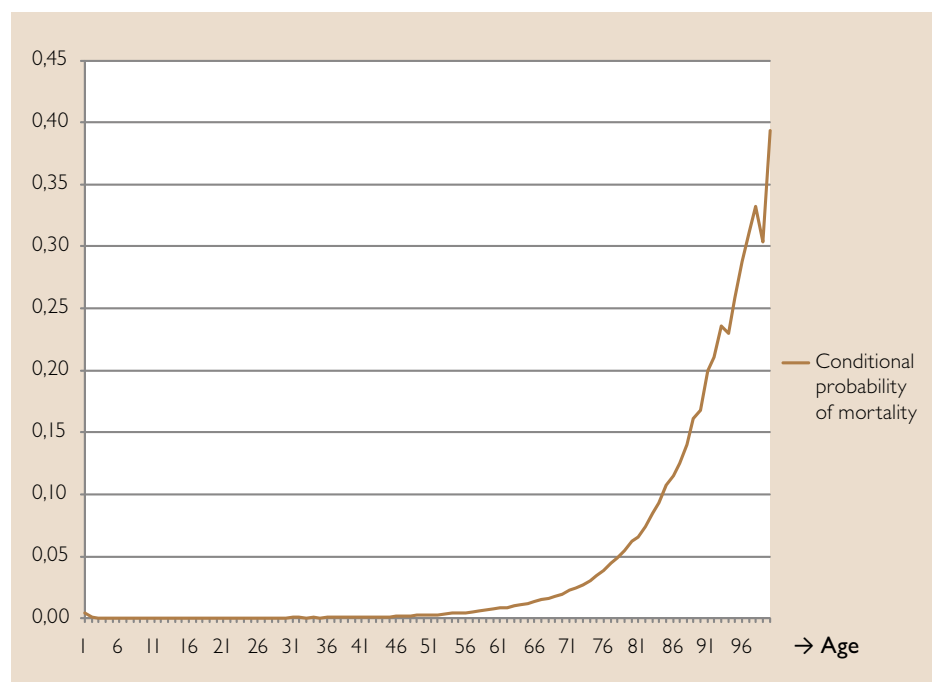


FIGURE 1. Conditional probability that a man deceases.

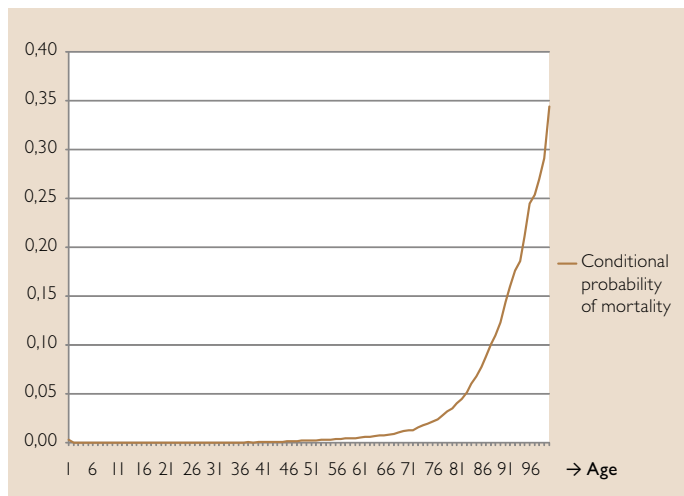


FIGURE 2. Conditional probability that a woman deceases.

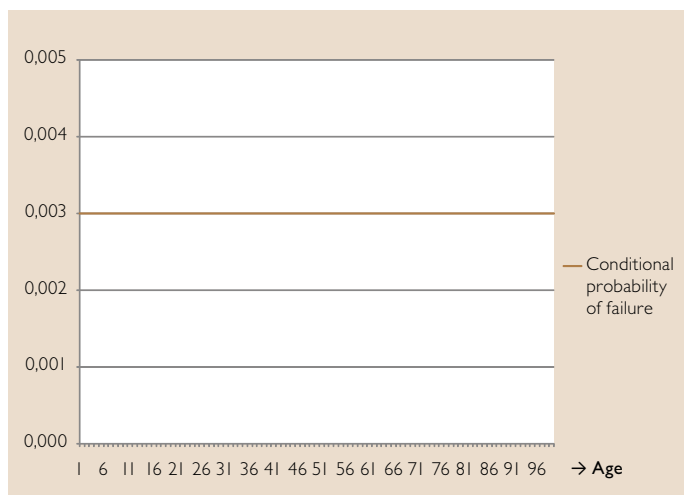


FIGURE 3. Conditional probability that a windshield breaks due to road debris.

ment strategy if the conditional probability of failure does not increase with age.

\* If  $E_1$  and  $E_2$  are two events, the probability that event  $E_2$  occurs given that event  $E_1$  has occurred, is known as the conditional probability of  $E_2$  given that event  $E_1$  has occurred. The conditional probability of a failure is the probability that the failure occurs in a specific period (e.g. year) given that the failure has not occurred until that period (e.g. year).

### Age-Related Failures

FIGURE 1 is based on the survival tables for men for the year 2007 as published by Statistics Netherlands. It shows the conditional probability of mortality. From a sample of 100.000 new-born boys, after 80 years there are 54.168 survivors and after 81 years 50.596. This means that, for men in their 81st year, the conditional probability of mortality is  $3.572/54.168 = 0,066$  (6,6 %).

FIGURE 2 is based on the survival tables for women for the year 2007. From a sample of 100.000 new-born girls, after 80 years there are 69.520 survivors and after 81 years 66.731. This means that, for women in

their 81st year, the conditional probability of mortality is  $2.789/69.520 = 0,040$  (4,0 %).

Figure 1 and Figure 2 show that the conditional probability of mortality is age-related, since it clearly increases with age.

### Random Failures

Imagine that each year on average three out of thousand windshields break because they are struck by road debris. In that case, from a sample of 1.000.000 windshields, after one year 997.000 are still intact, after two years 994.000, after three years 991.027 etc.

FIGURE 3 shows the conditional probability of a windshield breaking due to road debris. The probability that a new windshield breaks in the first year, is  $3.000/1.000.000$  or 0,003 (0,3 %). The probability that a windshield that survived the first year breaks in the second year, is  $2.991/997.000$  or 0,3 %. The probability that a windshield that also survived the second year breaks in the third year, is  $2.982/994.009$  or (again) 0,3 %.

The probability that a windshield that also survived the third year breaks in the fourth year, is  $2.973/991.027$  or (still) 0,3 % etc. This means that the conditional probability of failure is a constant. It is not in any way related to the age of the windshield. Such failures are known as random failures.

### The Role of Statistics

From our experience all of us will immediately be convinced that the conditional probability of mortality is age-related. Likewise, the conditional probability of a windshield breaking due to road debris being constant, will not come as a surprise to most of us. In these cases we do not need statistics to help us decide.

In practice however, for most failures it is not so obvious. In these cases we would like statistics to help us distil this information from technical history data.

One way of doing this is to plot the conditional probability of failure against the age, as in Figures 1, 2 and 3. This plot shows whether the failure is age-related or not. In their report, Nowlan and Heap describe six distinct patterns for such plots, as shown in FIGURE 4. Failures that conform to patterns A, B or C are age-related, while failures that conform to patterns D, E or F are not. Failures that conform to pattern E are random failures.

A different way to determine whether a failure is age-related or not, is to use one of the tests statisticians specifically designed for this purpose. Henk Tijms defines statistics as a science that seeks to derive conclusions that are valid on a wider scale, from limited data. This is exactly the situation that we are in. We have limited data that is data about the occurrences of a failure in the past. And we do indeed seek to derive conclusions that are valid on a wider scale that is conclusions about the occurrence of the failure in the future. The field of statistics offers a variety of models for describing observations obtained in experiments.

### The Normal Probability Distribution

TABLE 1 is based on an example in John Moubray's book. It shows technical history data about a sample of 100 pump impellers failing due to wear. All impellers lasted for at least 7 years. After 8 years, 2 were worn. After 9 years 16, after 10 years 50, after 11 years 84 and after 12 years 98. None of the

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impellers lasted for more than 13 years. Most impellers lasted for 10 or 11 years.

FIGURE 5 shows the conditional probability of a pump impeller failing due to wear. The field of statistics offers an excellent model for describing this impeller wear: the normal probability distribution. In other words, the time to impeller failure conforms to a normal probability distribution.

The normal probability distribution is characterized by two parameters: the mean  $\mu$  and the standard deviation  $\sigma$ . The normal probability distribution is symmetric around its mean. In our impeller example, the mean is 10 years. The mean defines how long the impellers last on average, in other words the mean time between failures. The standard deviation follows the deviations from the mean. In our impeller example, the standard deviation is approximately 1 year. This means that about 95 % of the impellers last between 8 and 12 years. About 2,5 % last shorter than 8 years and about 2,5 % last longer than 12 years. About 99,8 % of the impellers last for at least 7 years.

This means that we might select discarding the impeller every 7 years as our failure management strategy. Setting the interval for this scheduled discard task is an optimization problem and outside the scope of this paper.

### The Exponential Probability Distribution

TABLE 2 is also based on an example in John Moubray's book. It shows technical history data about a sample of 100 ball bearings failing due to fatigue.

FIGURE 6 shows the conditional probability of a ball bearing failing due to fatigue. Again, the field of statistics offers an excellent model for describing this bearing fatigue: the exponential probability distribution. In other words, the time to bearing failure conforms to an exponential probability distribution. A typical property of this probability distribution is the conditional probability of failure being constant. The exponential probability distribution is characterized by only one parameter. This parameter has the symbol  $\lambda$ . It corresponds to the conditional probability of failure.

Imagine a sample of 100 ball bearings. Hank selects discarding the ball bearings every 8 years as his strategy for managing fatigue. Fred decides to keep his hands of the ball bearings. Who is right? After 8 years, Hank would have 43 bearings left. He would then replace these 43 bearings by new ones. After 1 year he would have 39 bearings left, after 2 years 35, after 3 years 31, after 4 years 28, after 5 years 25, after 6 years 23, after 7 years 21 and after 8 years 19.

So, after 16 years in total, Hank would have 19 bearings left. And what about Fred? After 8 years, Fred would also have 43 bearings left. He would however not replace these bearings. How many bearings would Fred have left after another 8 years? Also 19! And if we had chosen an interval different than 8 years? Fred would still be right. Be-

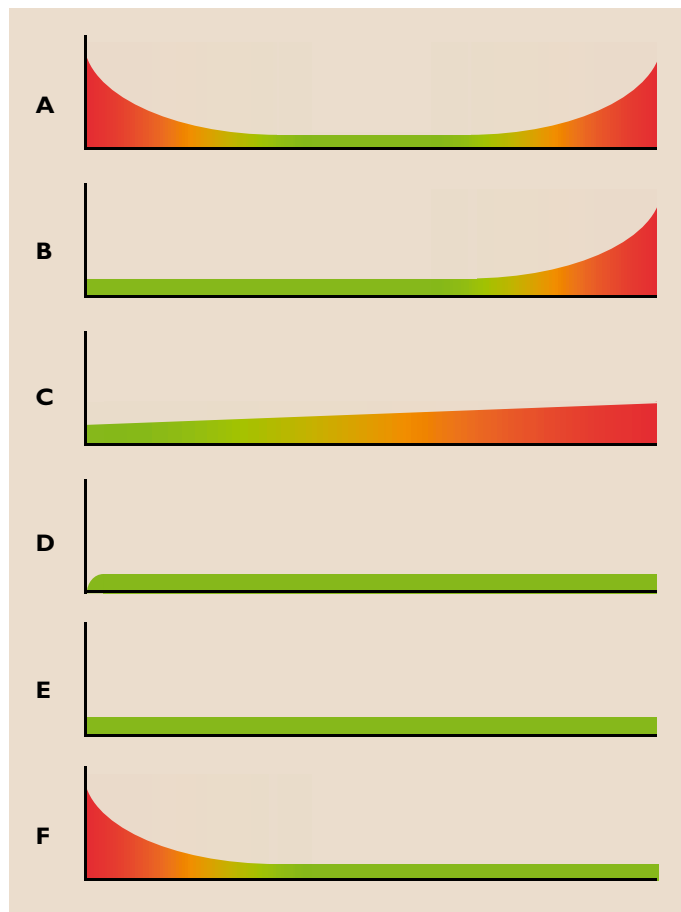


FIGURE 4. Six failure patterns.

cause the conditional probability of failure is constant, it makes no sense to discard the bearings at fixed intervals. If this is hard to believe, please remember the example of the windshields breaking due to road debris.

### The Weibull Probability Distribution

Besides the normal probability distribution and the exponential probability distribution, a third model is often used for describing failures, for instance by Lewis: the Weibull probability

distribution. The Weibull probability distribution is characterized by two parameters. One parameter has the symbol 'm' and is known as the shape parameter. The other parameter has the symbol  $\Theta$  and is known as the scale parameter.

The Weibull probability distribution includes both the normal probability distribution and the exponential probability distribution. Setting the shape parameter to a value of 1, will transform the Weibull probability distribution into the expo-

TABLE 1. Impeller wear.

YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13
Impellers before year	100	100	100	100	100	100	100	100	98	84	50	16	2
Impellers failed due to wear	0	0	0	0	0	0	0	2	14	34	34	14	2
Impellers after year	100	100	100	100	100	100	100	98	84	50	16	2	0
Conditional probability of failure	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,14	0,40	0,68	0,88	1,00

TABLE 2. Bearing fatigue.

YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Bearings before year	100	90	81	73	66	59	53	48	43	39	35	31	28	25	23	21
Bearings failed due to fatigue	10	9	8	7	7	6	5	5	4	4	4	3	3	2	2	2
Bearings after year	90	81	73	66	59	53	48	43	39	35	31	28	25	23	21	19
Conditional probability of failure	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10

nential probability distribution. And setting the shape parameter to a value larger than 2, will transform the Weibull probability distribution into the normal probability distribution. TABLE 3 shows the relation between the Weibull probability distribution on one hand and the normal probability distribution, the exponential probability distribution and the failure patterns on the other hand.

Failures that conform to a Weibull probability distribution

with  $m > 1$  are age-related failures. Failures that conform to a Weibull probability distribution with  $m = 1$  are random failures. For failures that conform to a Weibull probability distribution with  $m < 1$ , the conditional probability of failure decreases instead of increases with age. Usually, the strategy for managing this type of failures is a one-time change to the design, to the operating conditions, to a procedure or instruction or to knowledge and skills.

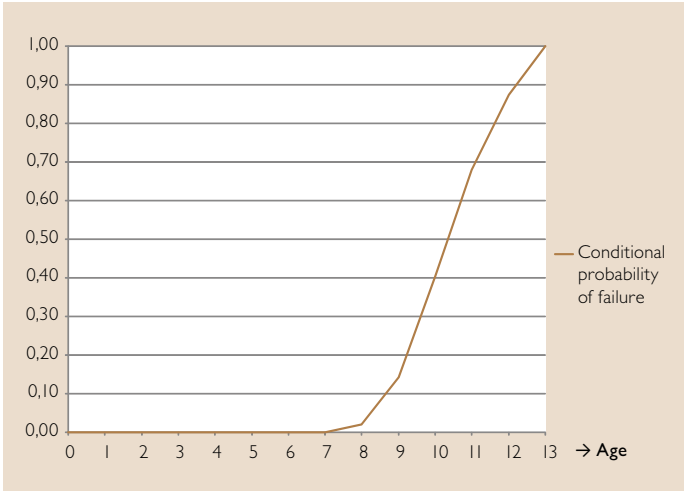


FIGURE 5. Conditional probability that an impeller fails due to wear.

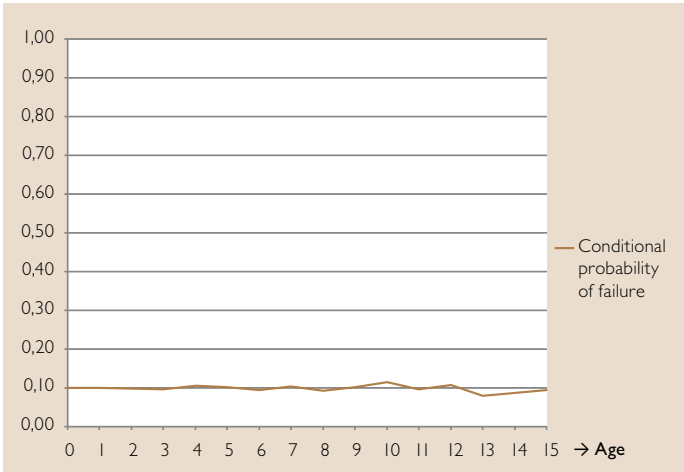


FIGURE 6. Conditional probability that a bearing fails due to fatigue.

Postulating Hypotheses – a Philosophical Approach

In our attempts to describe a failure, which probability distribution should be our starting point? In other words, what should we assume? That the time to failure conforms to a normal probability distribution, to an exponential probability distribution, to a Weibull probability distribution or to one of the many other probability distributions known from the field of statistics?

We could for instance try to describe the failures of the pump impellers due to wear discussed earlier, with an exponential probability distribution. That attempt would fail. Likewise, we could try to describe the failures of the ball bearings due to fatigue, with a normal probability distribution. That attempt would fail too.

In this case the technical history data are artificial and chosen with great care. In fact they are probably too good to be true. In practice technical history data are not that good and all three probability distributions discussed in this article could be used. So the question remains unanswered. Where do we start?

According to the philosopher Karl Popper, there are two types of theories:

- theories that are known to be wrong, as they were tested and adequately rejected (falsified),
- theories that have not yet been known to be wrong, not falsified yet, but are exposed to be proved wrong.

An interesting problem that has challenged many philosophers is that of the black swan. It is impossible to prove that all swans are white. Regardless of how many white swans we have seen, sooner or later we may bounce into a black swan. >>>



FIGURE 7. The black swan problem.

TABLE 3. Relation between probability distribution and failure patterns.

WEIBULL PROBABILITY DISTRIBUTION SHAPE PARAMETER	CORRESPONDING PROBABILITY DISTRIBUTION	CONDITIONAL PROBABILITY OF FAILURE	FAILURE PATTERN (ACCORDING TO NOWLAN AND HEAP)
$m > 4$	Normal	Increasing	B
$2 < m < 4$	Normal	Increasing	C
$1 < m < 2$		Increasing	D
$m = 1$	Exponential	Constant	E
$m < 1$		Decreasing	F

## WE SHOULD FIRST SELECT RANDOM FAILURE AND THE EXPONENTIAL PROBABILITY DISTRIBUTION AS THE NULL HYPOTHESIS.

This means that the theory that all swans are white is a theory that has not yet been known to be wrong, that has not been falsified yet, but that is exposed to be proved wrong. What if we try to prove the opposite that means that not all swans are white? This of course is perfectly possible. Seeing one single black swan would be enough to prove this theory. Popper explained that a theory can never be verified. Therefore we should postulate theories in such a way that they can be falsified.

In the field of statistics, Popper's theories are known as hypotheses. Popper suggests that any hypothesis should be postulated in such a way that we are able to prove it to be wrong (falsify it) using a test. Imagine for example that we want to prove that a pair of dice is loaded. In that case we would attempt to reject (falsify) the hypothesis that the pair of dice is fair. In the field of statistics this is known as the null hypothesis.

How should the null hypothesis read in the case of failures? That the failure is a random failure and conforms to the exponential probability distribution? Or that the failure is age-related and conforms to the normal probability distribution?

### Selecting the Null Hypothesis

Testing an hypothesis can lead to two types of errors. We reject the hypothesis when it should be accepted (type 1 error) or we accept the hypothesis when it should be rejected (type 2 error). Let us first assume that we select random failure and the exponential probability distribution as the null hypothesis. Imagine making a type 1 error, in other words rejecting the exponential probability distribution when it should be accepted. As

our next attempt, we should then select an age-related failure and the normal probability distribution as the null hypothesis. This hypothesis will almost certainly be rejected, so we would decide that scheduled discard is not a feasible failure management strategy and be compelled to consider alternatives. Now imagine making a type 2 error, in other words accepting the exponential probability distribution when it should be rejected. We would again decide that scheduled discard is not a feasible failure management strategy and consider alternatives.

Let us now assume that we select age-related failure and the normal probability distribution as the null hypothesis. Imagine making a type 1 error, in other words rejecting the normal probability distribution when it should be accepted. As before, we would decide that scheduled discard is not a feasible failure management strategy and be compelled to consider alternatives.

Finally, imagine making a type 2 error, in other words accepting the normal probability distribution when it should be rejected. We would decide that scheduled discard is a feasible failure management strategy, although the failure is not an age-related failure. The conditional probability of failure could be constant or even decrease with age. In the first case, scheduled discard will not reduce the conditional probability of failure, as shown by the example in section *The Exponential Probability Distribution*.

In the second case, scheduled discard would even be counter-

productive, since it would increase the conditional probability of failure. So, in case of this null hypothesis, a type 2 error would be delusive. We would be convinced that the failure is managed effectively by a scheduled discard task, while in fact it is not. In most cases this will not be acceptable. It will certainly not be acceptable in cases where the failure could effect safety or the environment.

The consequences of making a type 2 error in case of the second null hypothesis are far more serious than the consequences of making a type 1 or a type 2 error in case of the first null hypothesis. Hence, we should select random failure and the exponential probability distribution as the null hypothesis.

The following quote from Nassim Nicholas Taleb offers food for thought: "Shouldn't we be concerned with situations where patterns have been ignored, where non-randomness is mistaken for randomness? Mistaking non-randomness for randomness is not as costly as an error in the opposite direction. Even popular opinion warns that bad information is worse than no information at all". In the context of this paper Taleb points out that mistaking an age-related failure for a random failure is not as costly as mistaking a random failure for an age-related failure.

Finally, the number of parameters that characterize a probability distribution embodies a fundamental reason for selecting random failure and the exponential probability distribution as the null hypothesis. The exponential probability distribu-

tion only needs one parameter, whereas the normal probability distribution needs two parameters and other Weibull probability distributions also need two or even three. As a general rule, the more parameters a model has, the easier it becomes to fit that model to observations. However, adding more parameters makes the model more complex but not necessarily more realistic. Tests from the field of statistics do take the number of parameters required to characterize a probability distribution into account.

### Testing the Null Hypothesis

The following examples are discussed during The Aladon Network's introductory course on Reliability-centred Maintenance. Both examples are wonderfully in line with the following quote from John Allen Paulos: "Most quantities do not have nice bell-shaped distribution curves, and the average or mean value of these quantities is of limited importance without some measure of the variability of the distribution and an appreciation of the rough shape of the distribution curve".

The first example considers seizing of a ball bearing due to fatigue. Technical history data tell us that the first bearing seized after 3 years, the second bearing after 1 year and the third bearing after 5 years. The null hypothesis is that the failure is a random failure that conforms to the exponential probability distribution.

As described earlier, the exponential probability distribution is characterized by only one pa-



parameter. This parameter  $\lambda$  equals the conditional probability of failure. It also equals the reciprocal value of the mean time between failures. From these technical history data it follows that the mean time between failures is 3 years. This means that  $\lambda = 1/3$  per year. Then, for an exponential probability distribution with  $\lambda = 1/3$ , we derive the theoretical number of failures in the first or second year, in the third or fourth year, and in the fifth or sixth year. Finally, we compare the theoretical number of failures with the actual number of failures, for instance using the chi-square test. If these numbers differ too much, the hypothesis will be rejected.

In the case of this example the numbers do not differ enough for allowing the hypothesis to be rejected at a confidence level of 95 %. Not being able to reject the hypothesis, means that the failure may indeed be a random failure and that we should not assume that it is an age-related failure. Only for confidence levels below 73 %, the numbers differ enough and the hypothesis can be rejected. However, a confidence level of 73 % means that there is a 27 % chance of a type 1 error, in other words a 27 % chance that the hypothesis is rejected when it should be accepted.

The second example also considers seizing of a ball bearing

due to fatigue. Technical history data tell us that the first bearing seized after 3 years, the second bearing after 1 year and the third bearing after 6 years, the fourth bearing after 2 years, the fifth bearing after 5 years and the sixth bearing after 4 years.

From these technical history data it follows that the mean time between failures is 3,5 years. This means that  $\lambda = 2/7$  per year. Then, for an exponential probability distribution with  $\lambda = 2/7$ , we derive the theoretical number of failures in the first year, in the second year, in the third year, in the fourth year, in the fifth year and in the sixth year.

Again, we compare the theoretical number of failures with the actual number of failures, for instance using the chi-square test. Also in the case of this example the numbers do not differ enough for allowing the hypothesis to be rejected at a confidence level of 95 %.

Not being able to reject the hypothesis, means that the failure may indeed be a random failure and that we should not assume that it is an age-related failure. Only for confidence levels below 43 %, the numbers differ enough and the hypothesis can be rejected. However, a confidence level of 43 % means that there is a 57 % chance of a type 1 error, in other words a 57 % chance that the hypothesis is rejected when it should be accepted.

## Conclusions

In our efforts to select a probability distribution that adequately describes a specific failure, we should first select random failure and the exponential probability distribution as the null hypothesis. This hypothesis should be tested at a confidence level of 95 %, a level that is quite usual in the field of statistics. In case this null hypothesis cannot be rejected, a scheduled discard task is not a feasible failure management strategy. In case this null hypothesis can be rejected, age-related failure and the normal probability distribution should be selected as the next null hypothesis. In case this (second) null hypothesis cannot be rejected, a scheduled discard task could be a feasible failure management strategy. In case this (second) null hypothesis can be rejected, we should consider other failure management strategies. ■

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